Section 4.3
Equipping Your Function Toolkit

Curriculum Outcomes	Related Activities	Page in Text
• model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities • analyze and describe transformations of quadratic functions and apply them to absolute value functions • express transformations algebraically and with mapping rules • graph equations and inequalities and analyze graphs both with and without graphing technology • apply transformations when solving problems • use transformations to draw graphs	• four connected investigations that will explore parameter changes in the graphs of functions • an introduction to the absolute value function is given for those students not familiar with the absolute value function	174,176, 178,180 183

Transformations

Transformations are a form of math where we are comparing two different graphs to see how they have moved. There are 4 different types of transformations that we will be talking about in this unit.

In order to compare graphs, we must first graph our equations.

Create a table of values (using the same values as seen below) and graph each of the following.

1. $y = x^2$

2. $y = -x^2$

3. $y = x^2 + 3$

4. $y = x^2 - 6$

5. $y = 2x^2$

x	y
-3	9
-2	
-1	
0	
1	
2	
3	

(-3,9)

How to find the "y" in the table of values:

<u>Example</u>:

Using $y = x^2$

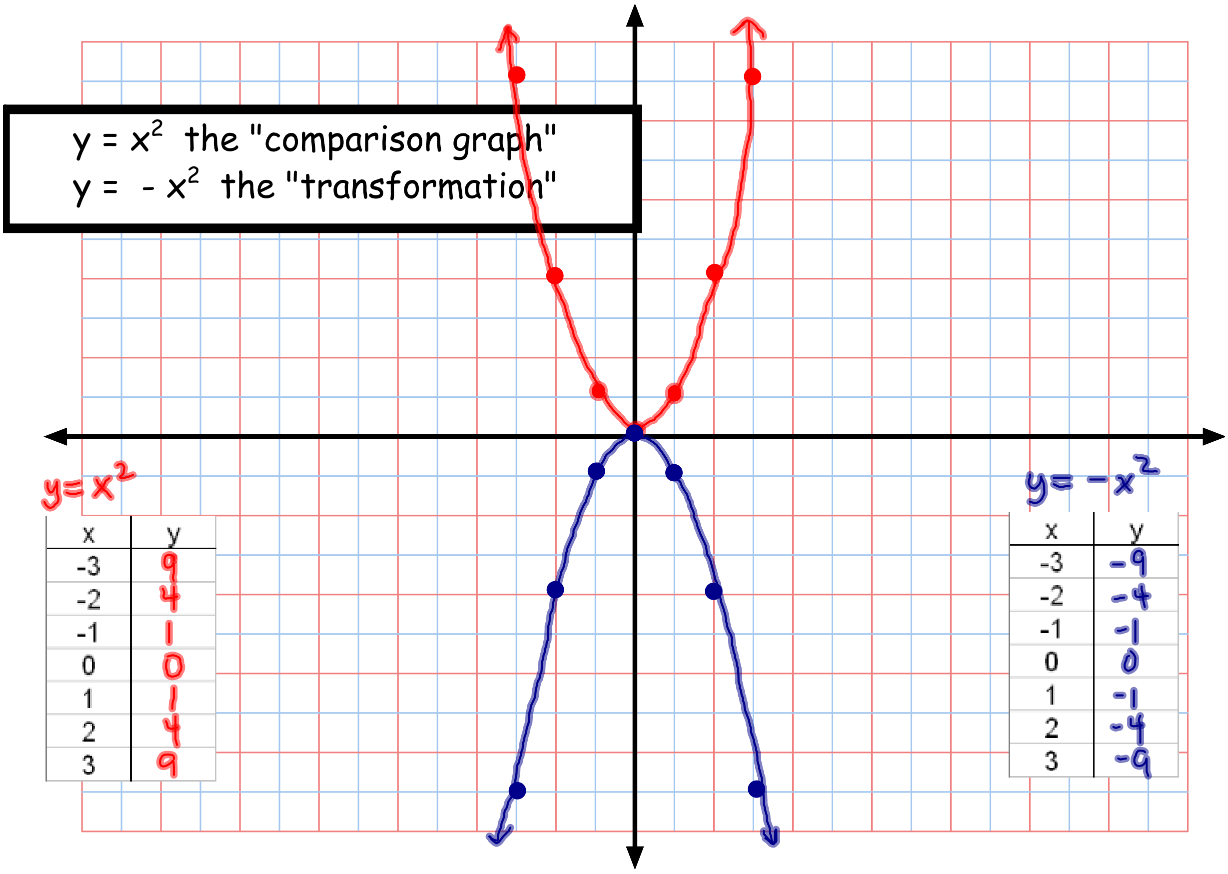
$y = x^2$
$y = (-3)^2$
$y = 9$

your first point to plot will be (-3,9)

Compare each graph to the $y=x^2$ graph

- $y = -x^2$ ---> what changed?
- $y = x^2 + 3$ ---> what changed?
- $y = x^2 - 6$ ---> what changed?

<u>For each comparison</u>, please copy down:
-what type of transformation it is
-the mapping notation.



$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = -x^2$

x	y
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9

$\boxed{y=-x^2}$

How do the 2 graphs compare?

We can describe how the 2 graphs compare verbally....

"Reflected" or "Flipped" over the x-axis.

And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"

y = x²

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

y = -x²

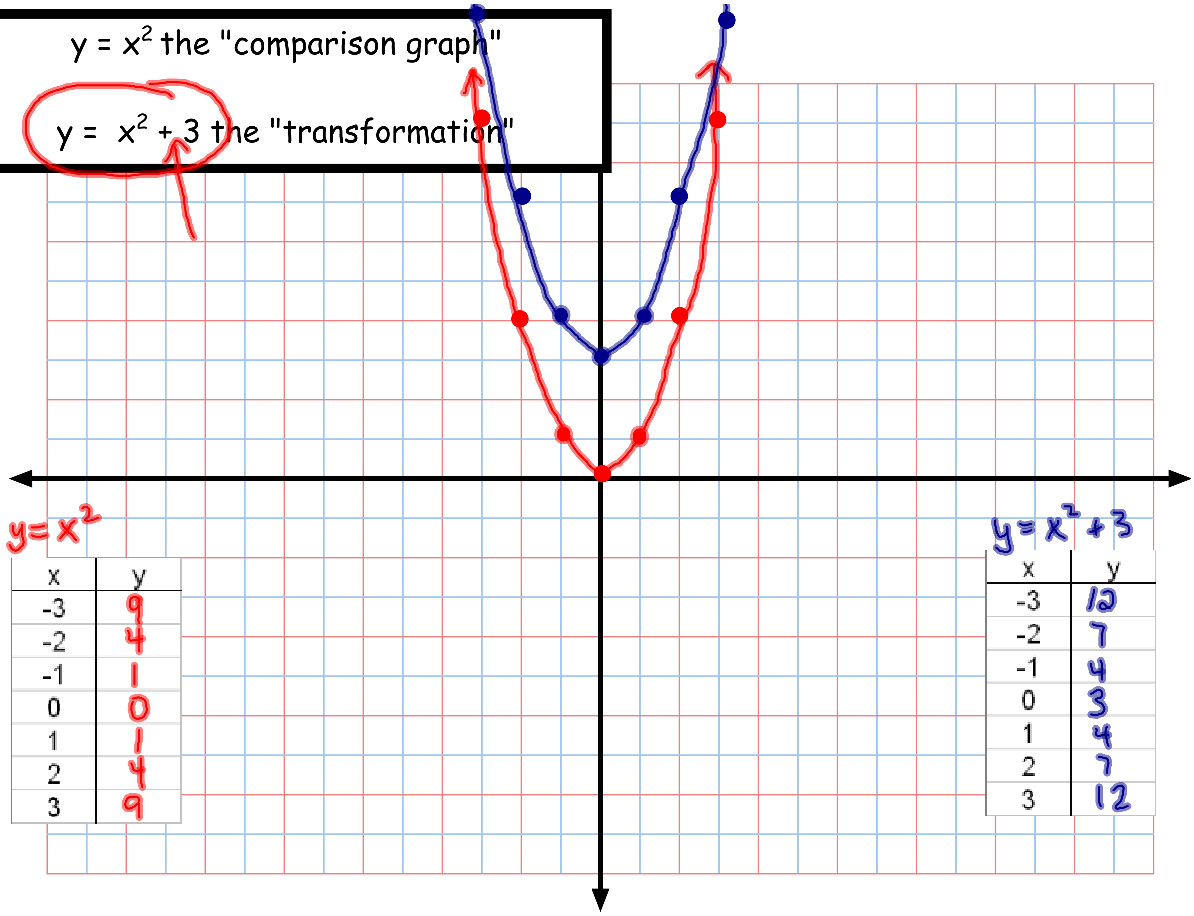
x	y
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9

$$\boxed{(x, y) \longrightarrow (x, -y)}$$

Notice:
Each y-value was multiplied by a negative.

$y = x^2$ the "comparison graph"

$y = x^2 + 3$ the "transformation"



$y=x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y=x^2+3$

x	y
-3	12
-2	7
-1	4
0	3
1	4
2	7
3	12

$y=x^2+3$

How do the 2 graphs compare?

We can describe how the 2 graphs compare verbally....

"Translation" or "slide" up 3 units

And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = x^2 + 3$

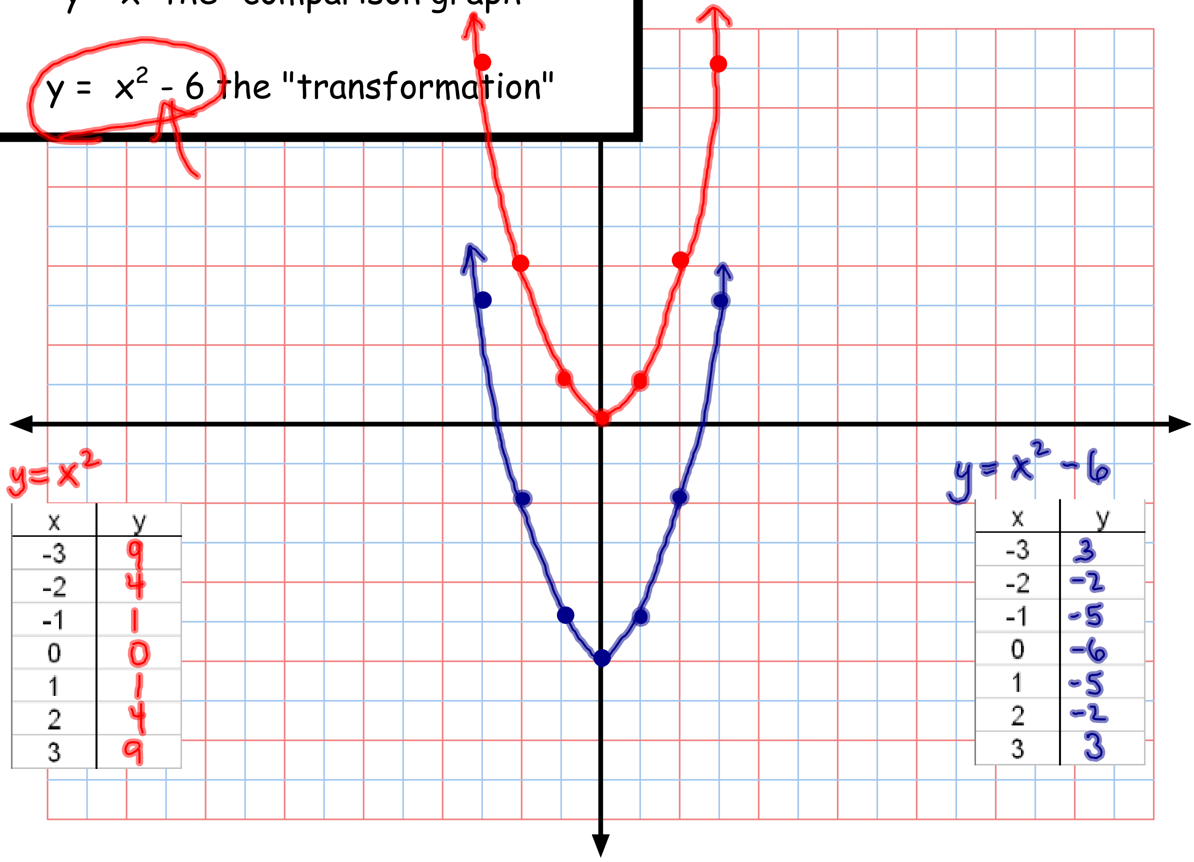
x	y
-3	12
-2	7
-1	4
0	3
1	4
2	7
3	12

Notice:
Each y-value had 3 added to it.

$$(x, y) \longrightarrow (x, y+3)$$

$y = x^2$ the "comparison graph"

$y = x^2 - 6$ the "transformation"



$y=x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y=x^2-6$

x	y
-3	3
-2	-2
-1	-5
0	-6
1	-5
2	-2
3	3

$y=x^2-6$

How do the 2 graphs compare?

We can describe how the 2 graphs compare verbally....

"Translation" or "slide" down 6 units

And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"

$y = x^2$

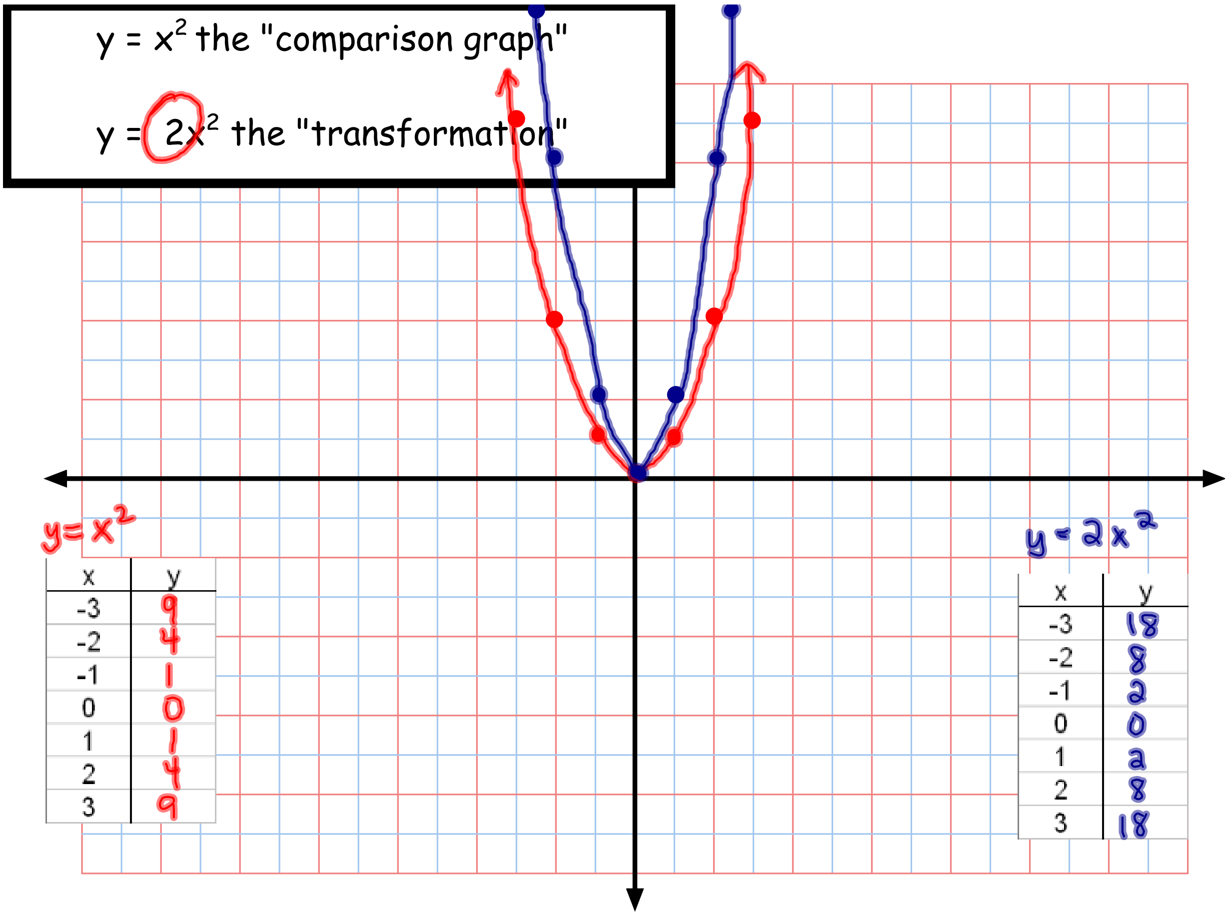
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = x^2 - 6$

x	y
-3	3
-2	-2
-1	-5
0	-6
1	-5
2	-2
3	3

Notice:
Each y-value had 6 subtracted from it.

$$(x, y) \longrightarrow (x, y-6)$$



$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = 2x^2$

x	y
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18

$y = 2x^2$

How do the 2 graphs compare?

We can describe how the 2 graphs compare verbally....

Vertical stretch of 2.
(2 times narrower)

And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = 2x^2$

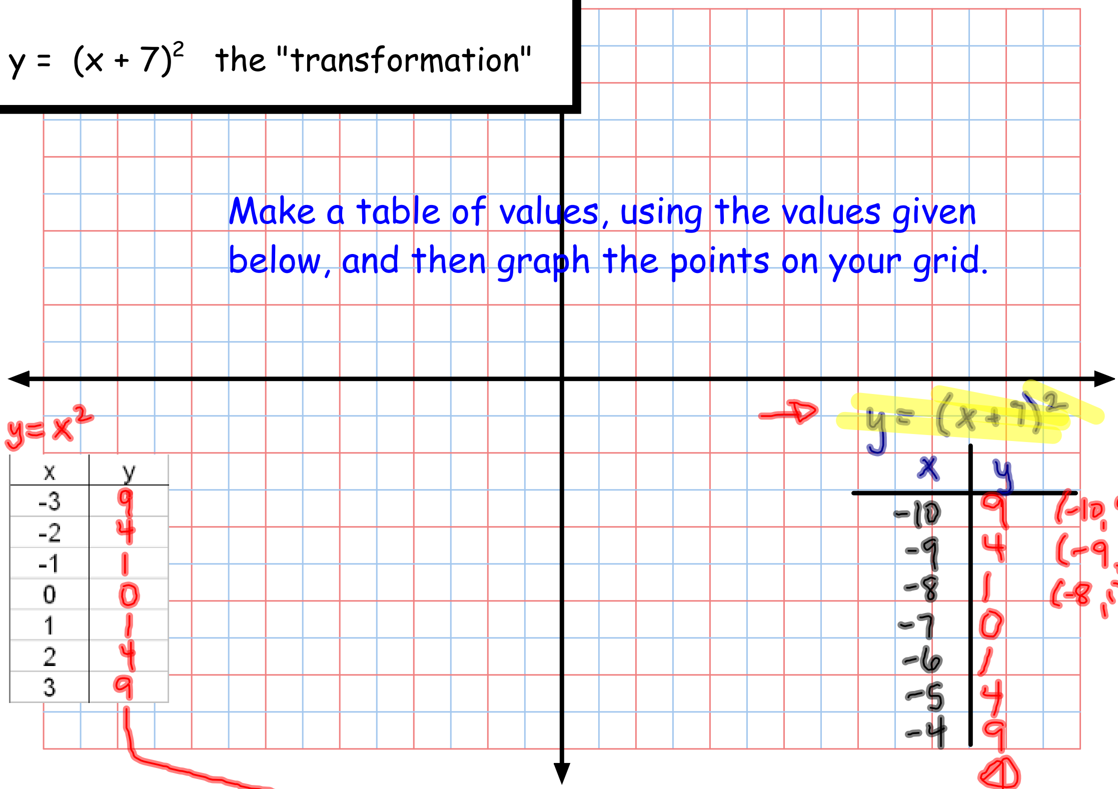
x	y
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18

Notice:
Each y-value doubled.

(x, y) $\longrightarrow (x, 2y)$

$y = x^2$ the "comparison graph"

$y = (x + 7)^2$ the "transformation"



Make a table of values, using the values given below, and then graph the points on your grid.

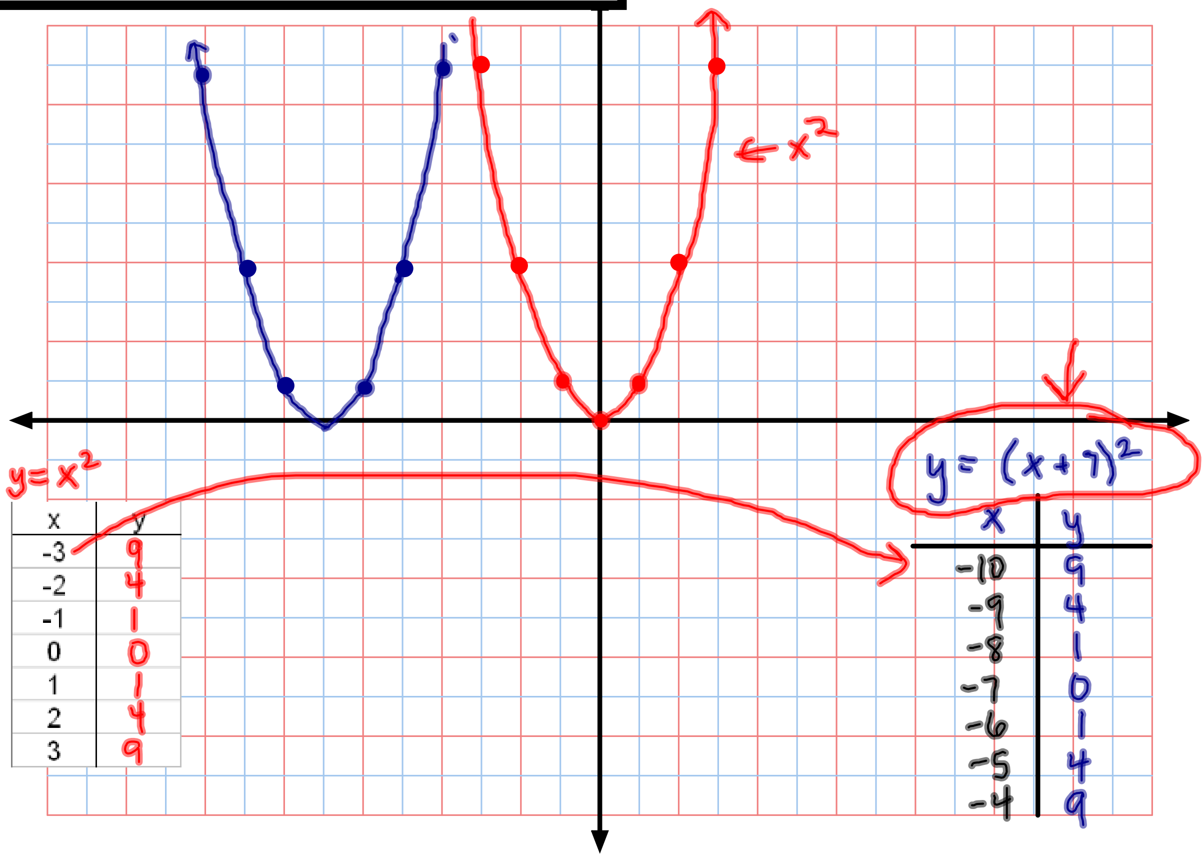
$y=x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y=(x+7)^2$

x	y	
-10	9	(-10, 9)
-9	4	(-9, 4)
-8	1	(-8, 1)
-7	0	
-6	1	
-5	4	
-4	9	

$y = (x + 7)^2$ the "transformation"



$y=x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y=(x+7)^2$

x	y
-10	9
-9	4
-8	1
-7	0
-6	1
-5	4
-4	9

$y=(x+7)^2$

How do the 2 graphs compare?

We can describe how the 2 graphs compare verbally....

"Translation" or "slide" to the left 7 units
Horizontal

And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = (x + 7)^2$

x	y
-10	9
-9	4
-8	1
-7	0
-6	1
-5	4
-4	9

$(x, y) \longrightarrow (x-7, y)$

Standard Form Vs. Transformational Form

The **standard form** of an equation is the form you are used to using

- it has y by itself

$y = -x^2$ **$y = -x^2 - 1$** **$y = x^2 + 8$**

The **transformational form** of an equation is a form that has the x^2 by itself

$y = -x^2$ **$y = -x^2 - 1$** **$y = x^2 + 8$**

If you are given an equation that is not in standard form, you will need to rearrange it so that it is.

This is necessary to create a table of values!

Notes to help you remember the types of transformations:

copy

1. Reflection
- this means that the graph flips
- the equation: $y = -x^2$
- the mapping rule: (x, y) -> (x , -y)

2. Vertical Stretch
- this means that the graph gets skinnier or wider
- the larger the V.S. value, the skinnier the graph
- the equation: $y = 2x^2$
- the mapping rule: (x, y) -> (x, 2y)
- this graph gets skinnier by a V.S. of 2

3. Vertical Translation
- this means that the graph moves up or down
- the equation: $y = x^2 + 5$
- the mapping rule: (x, y) -> (x, y + 5)
- this graph moves up 5

4. Horizontal Translation
- this means that the graph moves left or right
- the equation: $y = (x - 3)^2$
- the mapping rule: (x, y) -> (x + 3, y)
- this graph moves right 3 (always the opposite of what is in the equation)

$(x, y) \longrightarrow (x-2, -\frac{1}{3}y-8)$

Ex. $y = -\frac{1}{3}(x+2)^2 - 8$

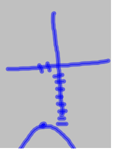
(3, 9) → (1, -11)

Reflection

V.S. of 1/3 (wider)

H.T. of Left 2

V.T. down 8



These are the ONLY transformations that you will see. The numbers for each transformation are ALWAYS located in the same place in the equation and in the mapping rule.

In order to determine and describe the type of transformation, you will need to rearrange the equation into standard form.

Example of how to rearrange an equation into the form y=

$$\frac{1}{4}\boxed{y} + 6 = (x+2)^2$$
$$-6 \qquad -6$$

$$\cancel{4} \cdot \left(\cancel{\tfrac{1}{4}} y\right) = \left((x+2)^2 - 6\right) \cdot 4$$

$$y = 4(x+2)^2 - 24$$

- H.T. of left 2
- V.T. of down 24
- V.S. of 4 (skinnier)